The volume of a cone

Let $D \subset \mathbb{R}^n$ be a set with n-volume $|D|$, and let $P = (a_1, \ldots, a_n, h) \in \mathbb{R}^{n+1}$ be a fixed point with $h > 0$. We consider that cone C consisting of all line segments from P to a point in D. The canoe can be parametrized with the variables $x_1, \ldots, x_n, t$, where $(x_1, \ldots, x_n) \in D$, $t \in [0, 1]$ and the equations

$$(y_1, \ldots, y_{n+1}) = (x_1, \ldots, x_n, 0) + t\,(x_1 - a_1, \ldots, x_n - a_n, -h)\ .$$

It is easy to see that

$$\left|\frac{\partial(y_1, \ldots, y_{n+1})}{\partial(x_1, \ldots, x_n, t)}\right| = ht^n\,,$$

and it follows that that $n+1$-volume of C is

$$|C| = \frac{1}{n+1}|D|h\,.$$